From reliable initialised forecasts to skilful climate projection

a dynamical systems approach

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How can we assess climate models?

- Is there a link between model performance on shorter timescales and that model's climate response?
  - Make use of short range forecasts, where verification is possible, to constrain climate projections?
  - Emergent constraint

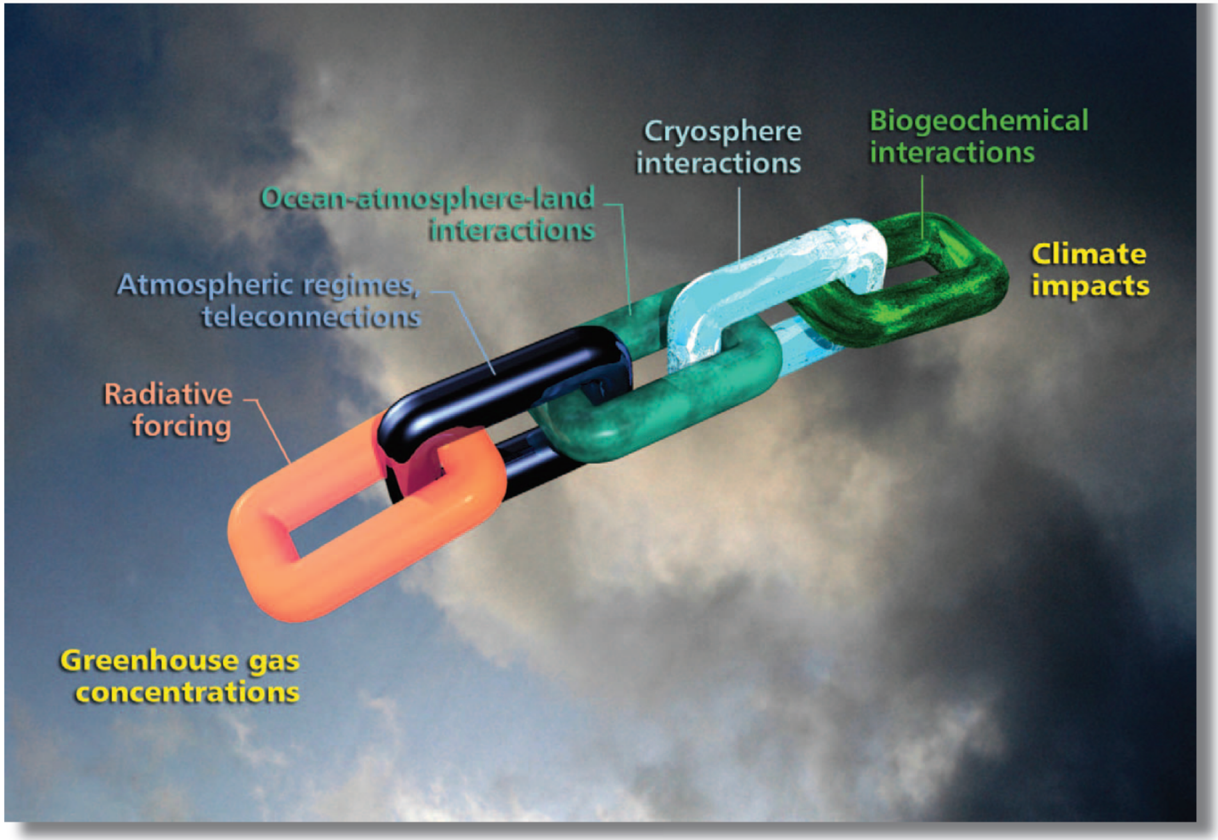
BAMS, 2008

TOWARD SEAMLESS PREDICTION

Calibration of Climate Change Projections Using Seasonal Forecasts

BY T. N. PALMER, F. J. DOBLAS-REYES, A. WEISHEIMER, AND M. J. RODWELL

In a seamless prediction system, the reliability of coupled climate model forecasts made on seasonal time scales can provide useful quantitative constraints for improving the trustworthiness of regional climate change projections.



But what is the basis for this statement?

- Is there a link between the reliability of initialised forecasts and the response of a system to an external forcing?

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Outline:

- Use a **dynamical systems theory** framework to explore whether there is a physical basis for this statement
- Test the predicted link between reliability and response to forcing using a simple atmospheric model

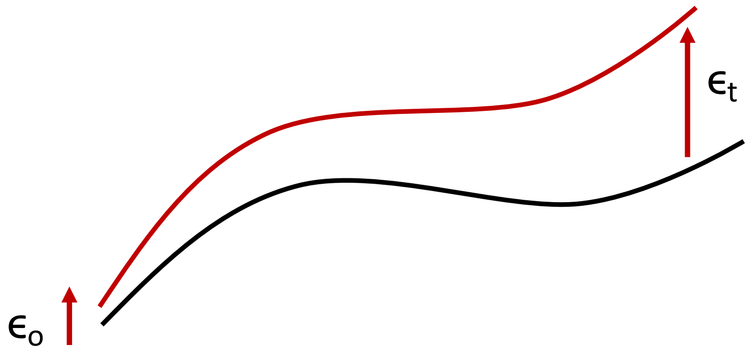
Dynamical systems

- We describe our dynamical system by a set of nonlinear differential equations

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$$

- Introduce the tangent propagator, which maps a small perturbation at t=0 onto a later time

$$\boldsymbol{\epsilon}_t = \mathbf{M}(t, t_0, \mathbf{x}_0)\boldsymbol{\epsilon}_0$$



Reliability in dynamical systems

- Reliable forecast:
  - How consistent is the forecast probability of an event with the measured probability?
  - Verification behaves as if drawn from the ensemble forecast

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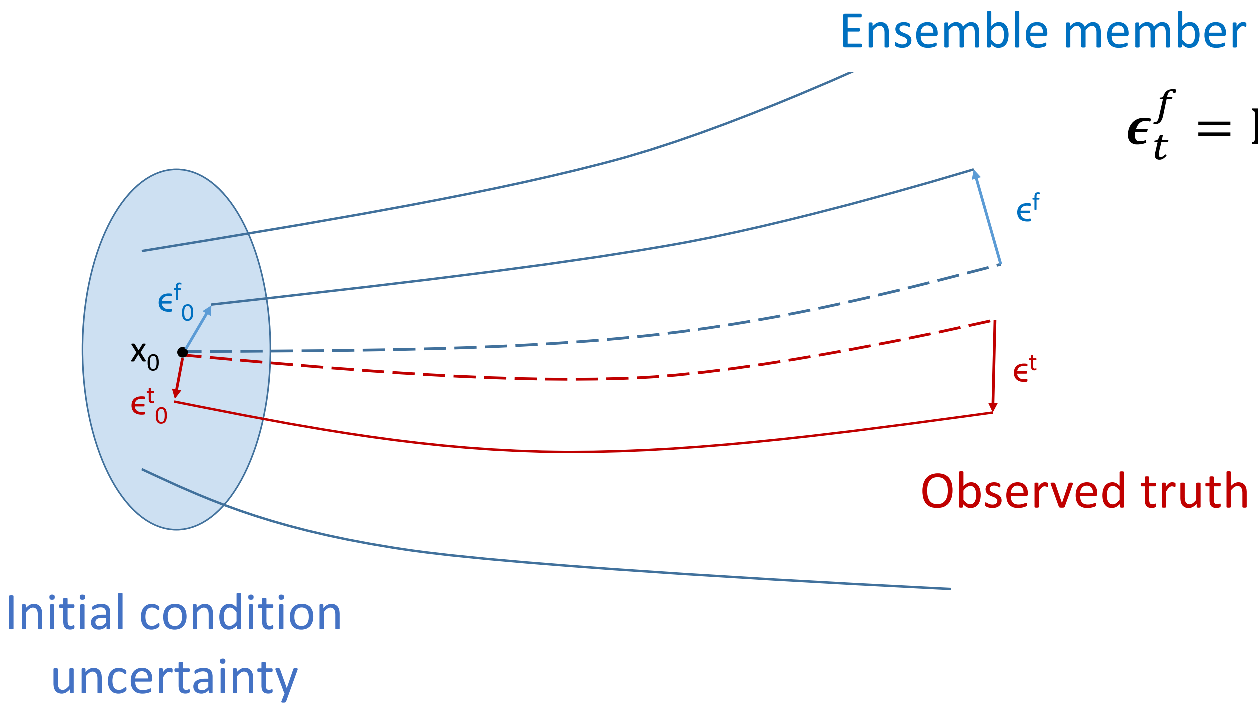

Response: predicted by linear response theory

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) \xrightarrow{\text{Apply small perturbation}} \dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) + \boldsymbol{\delta}\mathbf{F}$$

- Assume response, $\delta\mathrm{x}$, is linearly proportional to perturbation: $\delta\mathrm{x} \propto \delta\mathrm{F}$
- Follow Ruelle's formulation of linear response theory, suitable for chaotic systems

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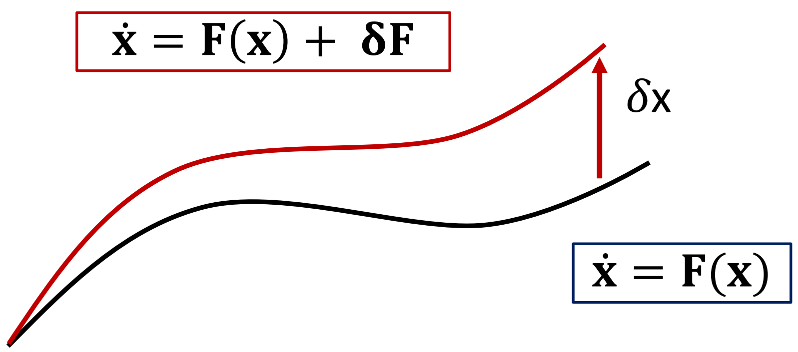
$$\delta\mathbf{x}(t_0, t) = \int_{t_0}^{t} \mathrm{d}t' \mathbf{M}(t, t', \mathbf{x}_{t'}) \delta\mathbf{F}(t', \mathbf{x}_{t'})$$



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- To calculate expected response, average over the attractor, integrate all perturbations from $t_o = -\infty$ to present, change variable of integration. Assuming $\mathrm{F(x)}$ and $\delta\mathrm{F(x)}$ independent of t:

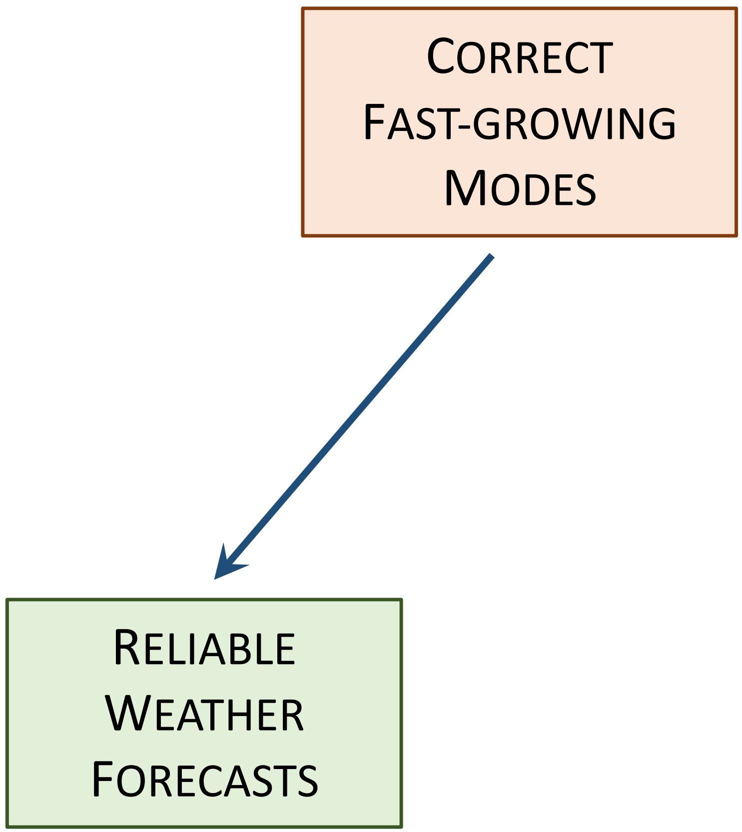
$$\delta\langle\mathbf{x}\rangle = \int_{0}^{\infty} \mathrm{d}\tau \overline{\mathbf{M}}(\tau) \delta\mathbf{F}$$

So what can we learn about seamless prediction?

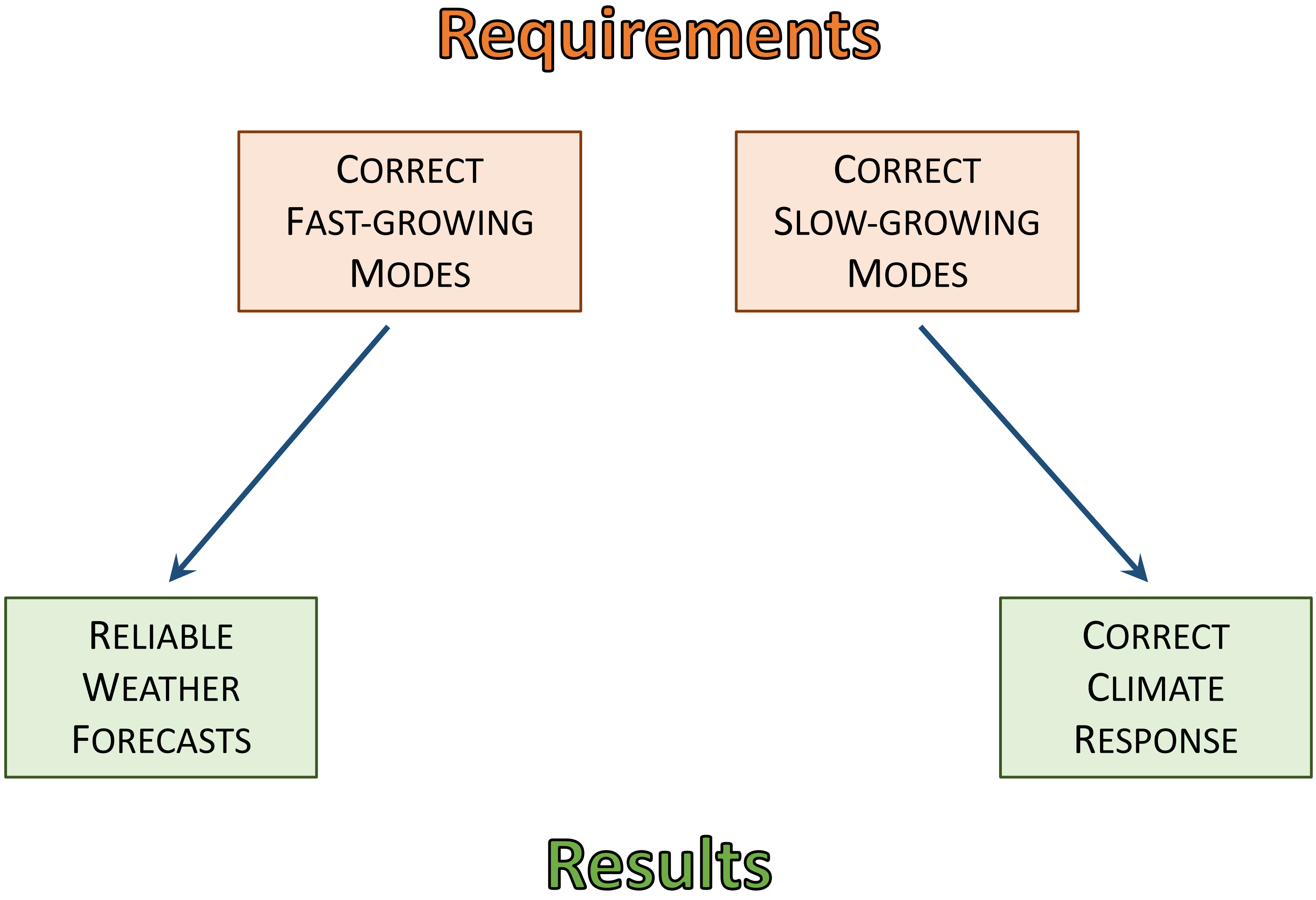
$$\delta\langle\mathbf{x}\rangle = \int_0^\infty \mathrm{d}\tau \bar{\mathbf{M}}(\tau)\delta\mathbf{F}$$

- Forecast models which correctly represent M will capture the response of the true system
- Reliability is a way of assessing M
- But, reliability primarily assesses fastest growing modes, whereas response requires accurate representation of M(τ) out to τ=∞
- M(τ) at short τ may look quite different to M(τ) at long τ, as different processes are important on different time scales

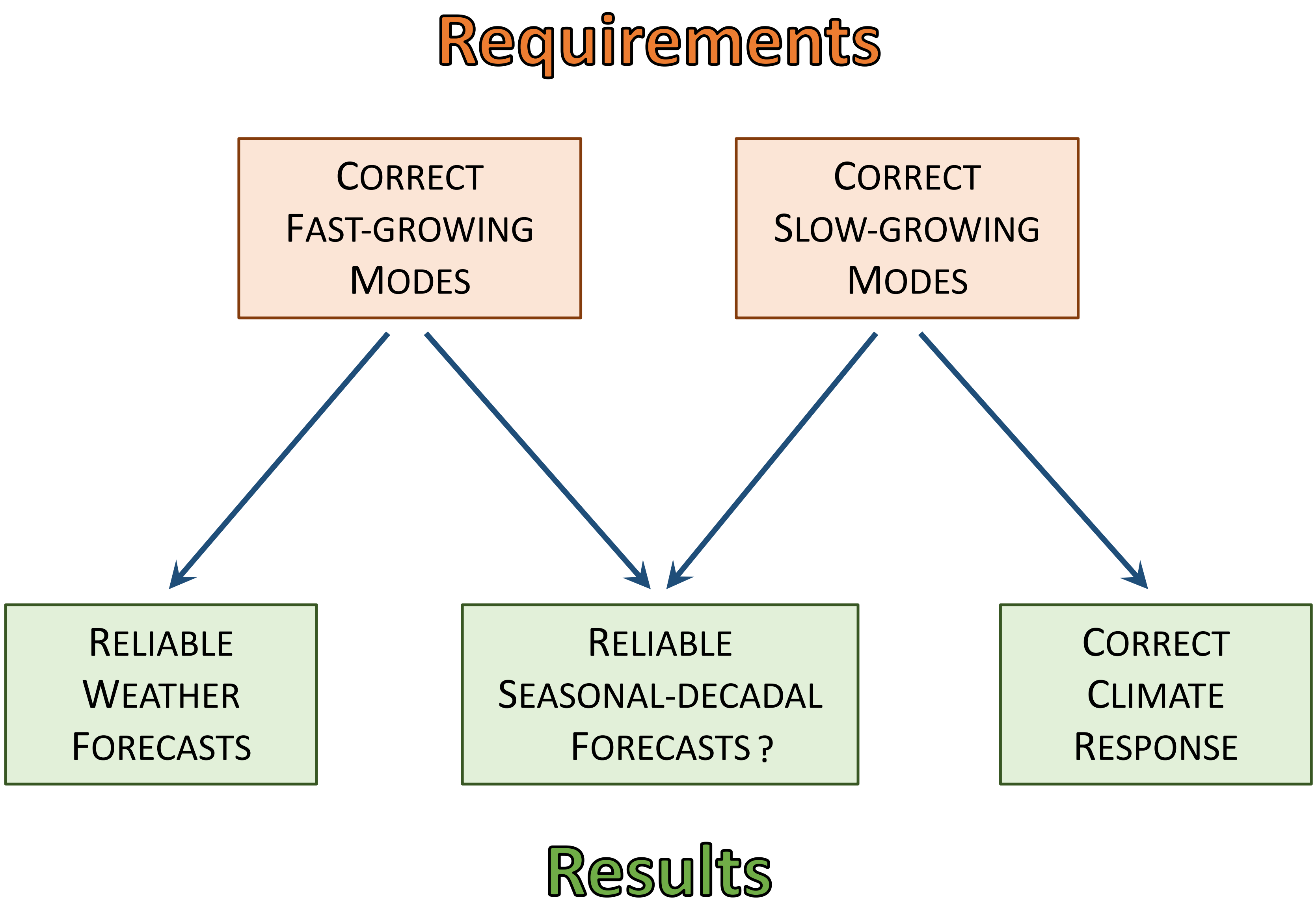
Requirements



Results



Requirements
Correct Fast-growing Modes
Correct Slow-growing Modes
Reliable Weather Forecasts
Correct Climate Response
Results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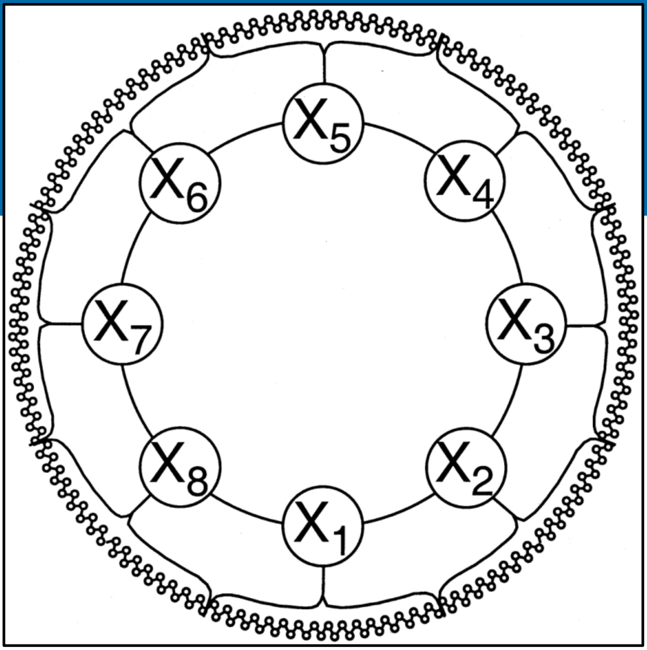
Correct Fast-growing Modes
Correct Slow-growing Modes
Reliable Weather Forecasts
Reliable Seasonal-decadal Forecasts ?
Correct Climate Response
Results

Can we test this theory?



- Use the Lorenz 96 system
  - Treat full model as our 'true' climate system
  - Perturb using an imposed forcing
  - Build forecast models of unperturbed system

$$\frac{dX_k}{dt} = -X_{k-1}\left(X_{k-2} - X_{k+1}\right) - X_k + F + \delta F_k - \frac{hc}{b}\sum_{j=J(k-1)+1}^{kJ} Y_j$$

$$\frac{dY_j}{dt} = -cbY_{j+1}\left(Y_{j+2} - Y_{j-1}\right) - cY_j + \frac{hc}{b}X_{\mathrm{int}[(j-1)/J]+1}$$

δF	(3,0,0,0,0,0,0,0)

Lorenz, 1996
Arnold et al, 2013

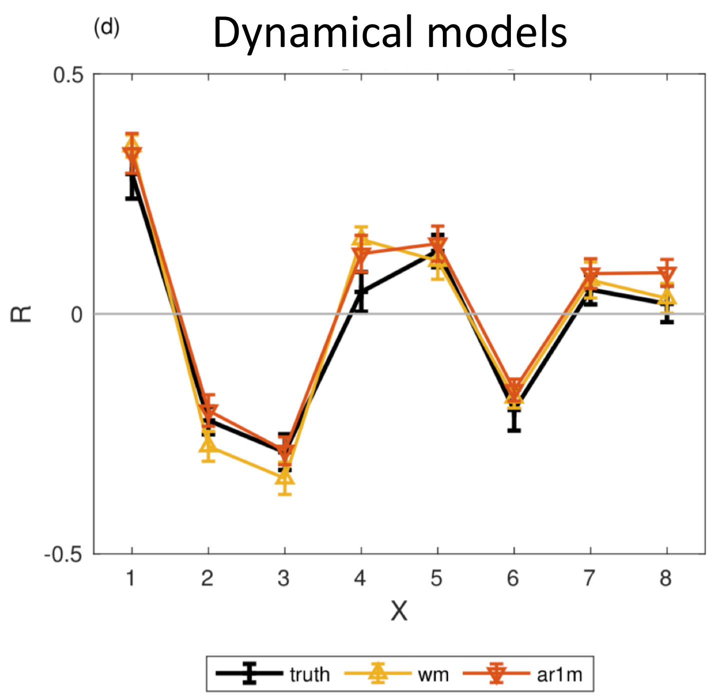
Forecast models

Dynamical model

- Assume we know the equations of motion, but cannot afford to compute the fast Y variables
- Replace Y with an additive stochastic parametrisation
- Test *white noise* – believed to be a poor model of sub-grid scales
- Test *red noise* – believed to be a good model of sub-grid scales

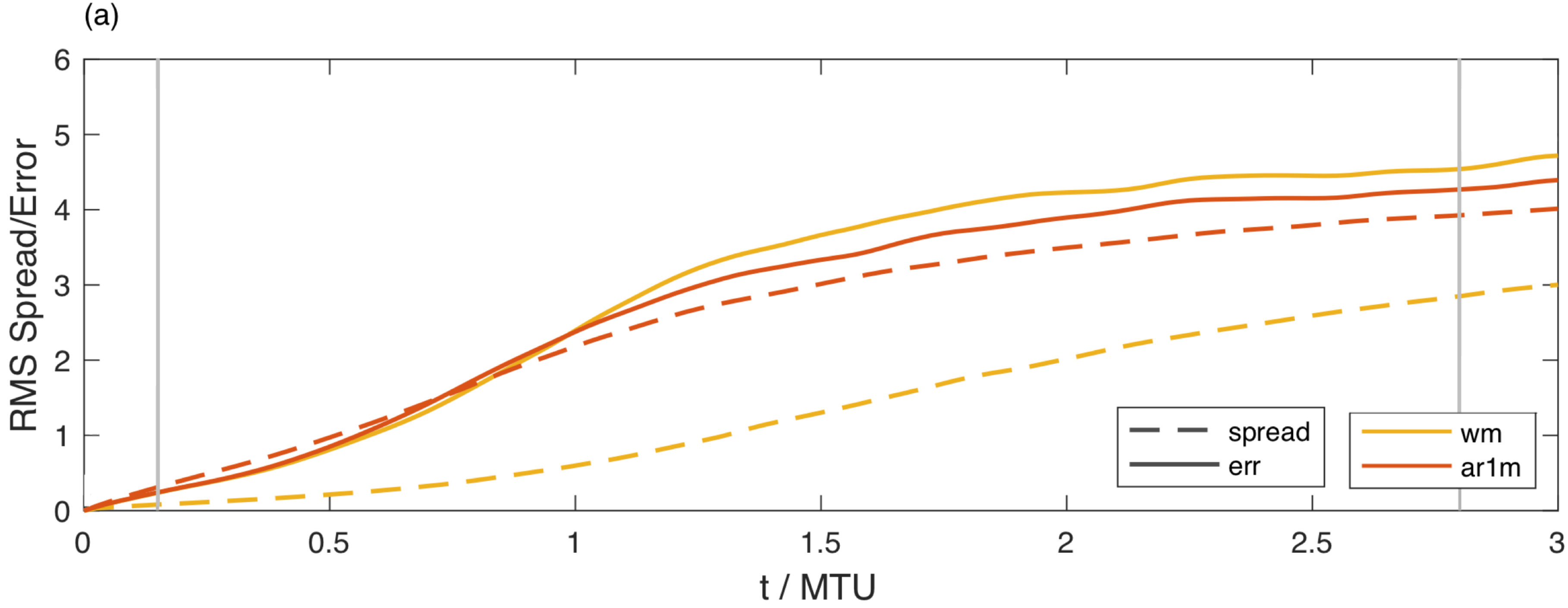
Response to forcing

- Run the full 'true' system with and without forcing perturbation
- Run each model of the system with and without forcing perturbation



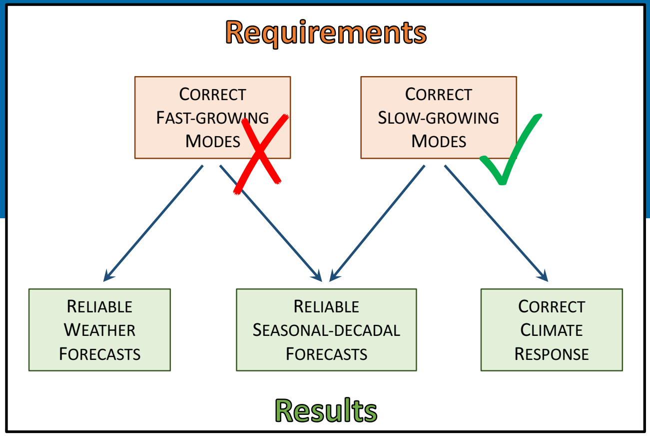
Now consider reliability

- Run 40-member ensemble forecasts of the unperturbed system from 300 start dates

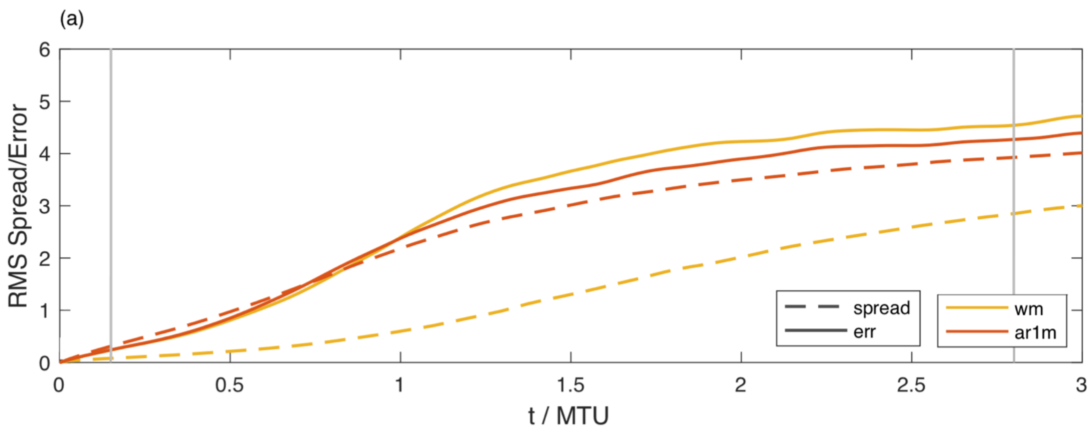


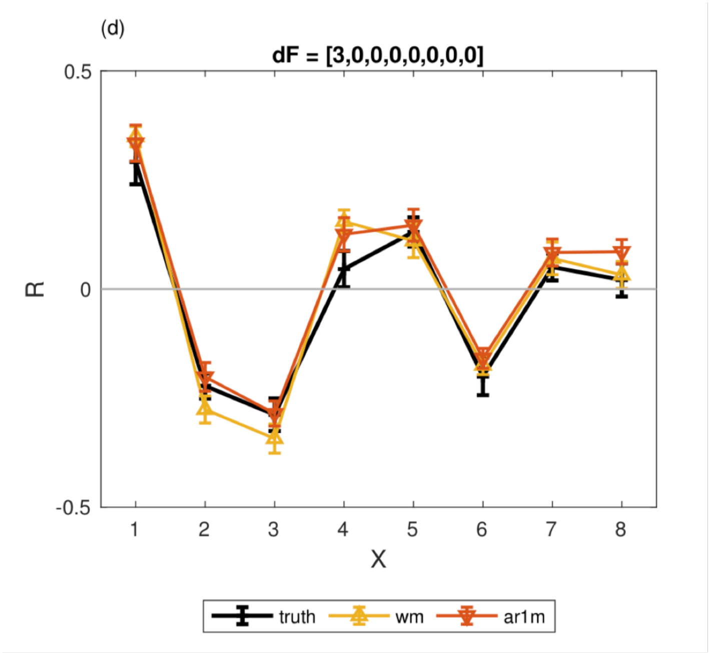
N.b. 3 MTU ~= 15 'days', which is about as long as we have predictability for in L96

Interpretation



- White noise
  - Is not reliable at any timescale
  - Has a good response to the forcing





Lesson in caution:
White noise poorly represents fastest scales, but we still have the L96 dynamics to correctly represent the slowest scales

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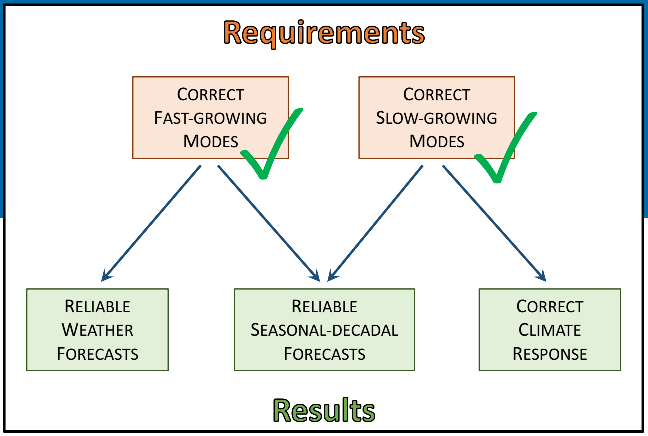
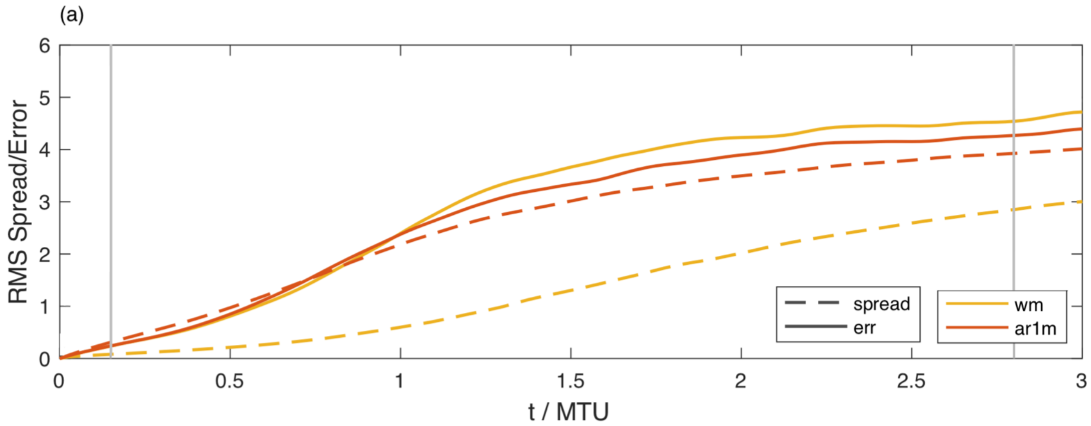
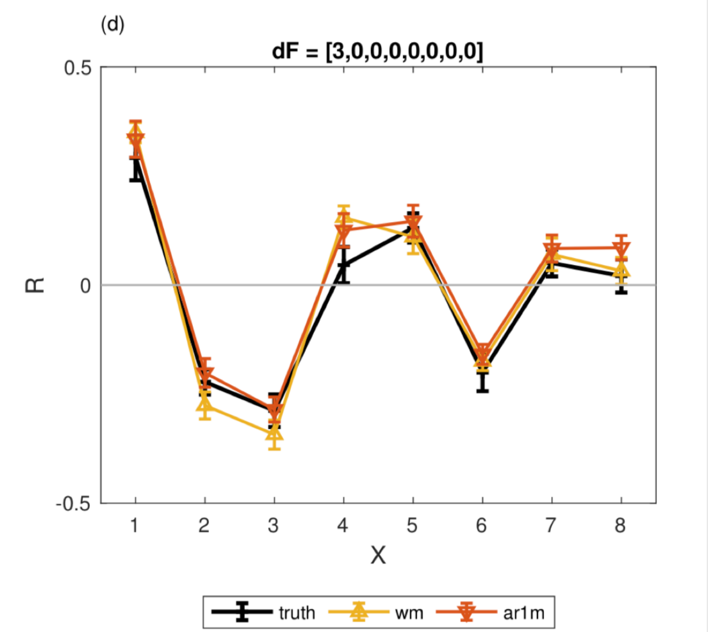
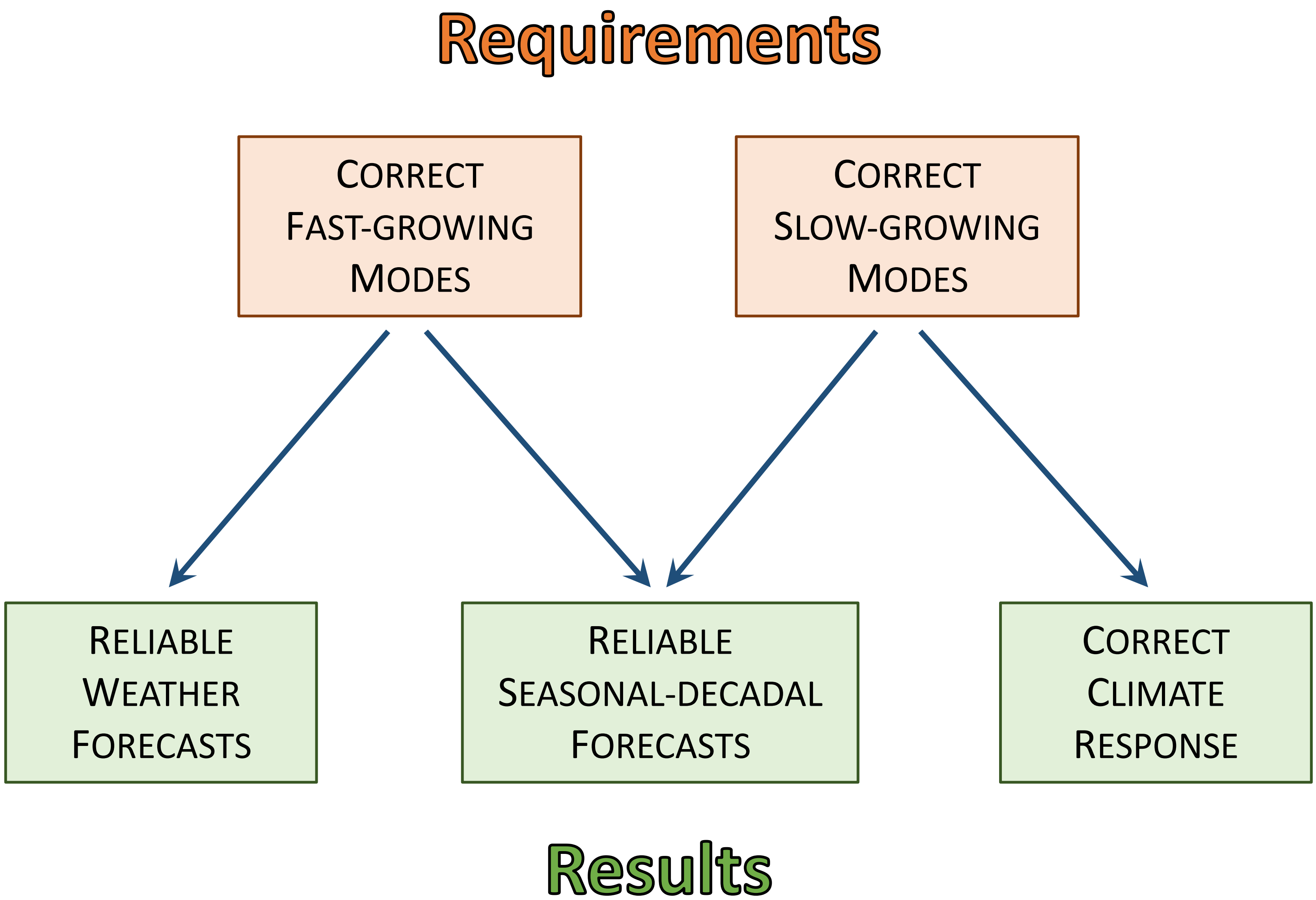


- Red noise
  - Seems reliable at *all* timescales
  - And has a good response to forcing!





Red noise captures the fastest scales well, and we still have the L96 dynamics to correctly represent the slowest scales



Christensen and Berner, *From reliable weather forecasts to skilful climate response: a dynamical systems approach.* Submitted to QJRMetS

Extra slides

Forecast models

1. Statistical model
   - Replace dynamical equations with a linear model fitted to the data, designed to get variance correct.
2. Dynamical model
   - Assume we know the equations of motion, but cannot afford to compute the fast Y variables
   - Replace Y with an additive stochastic parametrisation
   - Test *white noise* – believed to be a poor model of sub-grid scales
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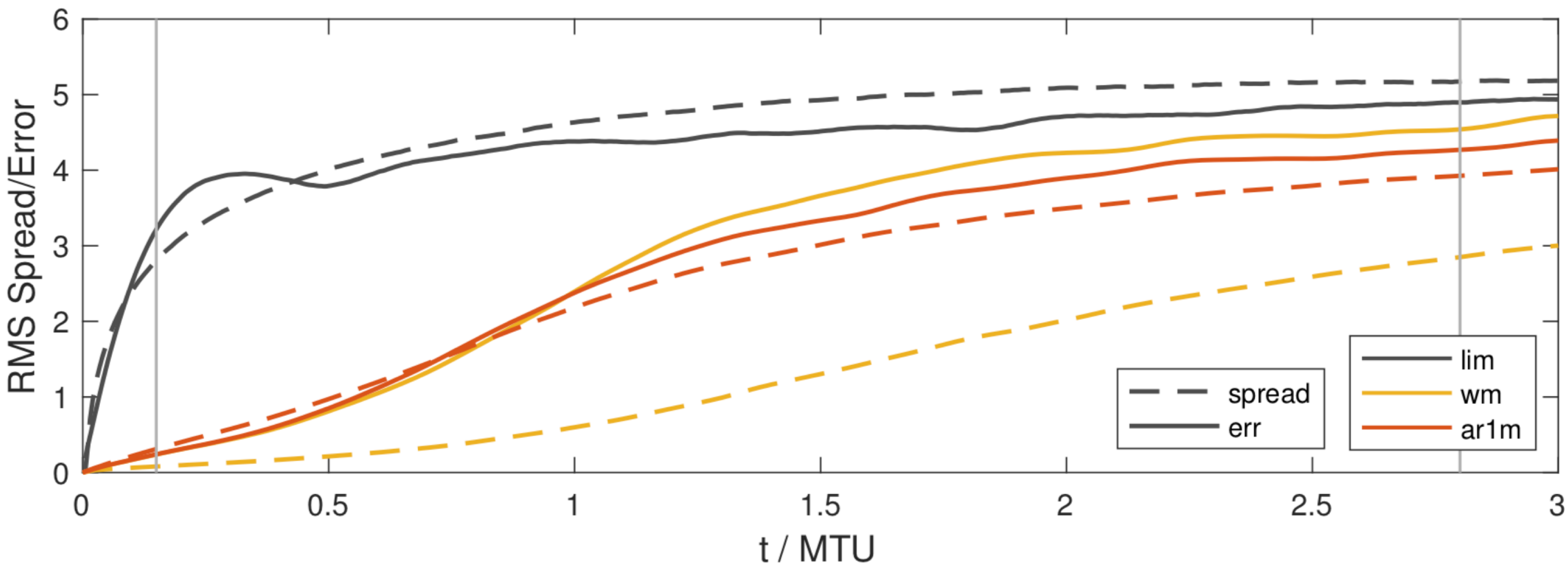

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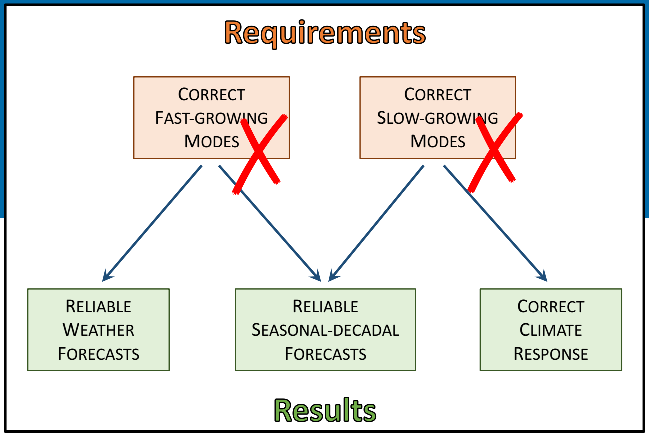
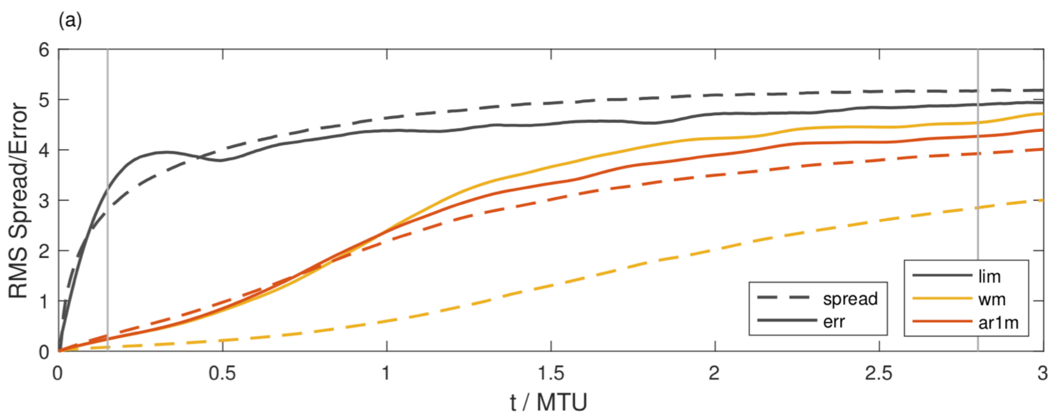
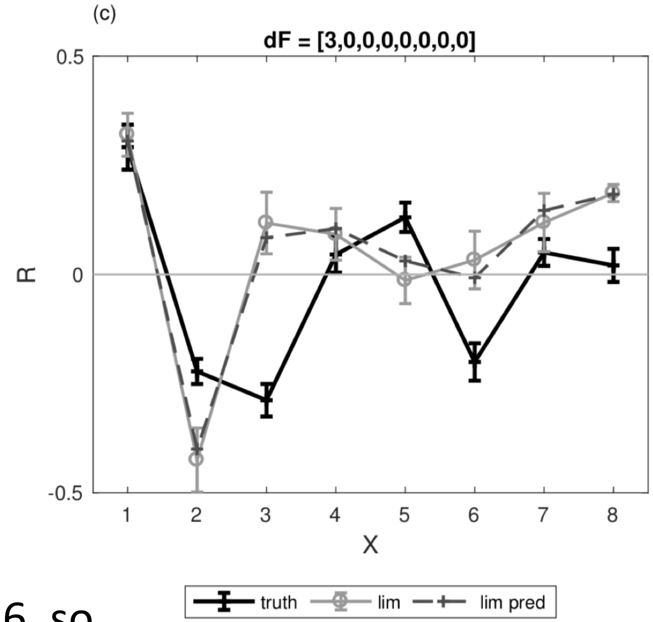


- Statistical model
  - Seems reliable at all timescales
  - But has poor response to forcing!





Statistical model is poor representation of dynamics of L96, so model error term is large. Reliability does not assess M_f