IMPROVING SPHERICAL HARMONIC COEFFICIENTS FOR THE FINE RESOLUTION NCEP GLOBAL ATMOSPHERIC SPECTRAL MODELING

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1. Introduction

At the Environmental Modeling Center, NCEP (National Centers for Environmental Prediction), we are working on meeting the challenge of adding cloud-resolvable and deep-atmospheric capabilities to the NCEP GFS (Global Forecast System), which is our global atmospheric spectral modeling system. To have a very fine resolution global spectral model, there are two major obstacles to conquer, based on the experiences of ECMWF (Wedi et al. 2012): one is the capability to do a spectral transform with thousands waves; the other is the speed up of the fast Legendre transform. There are potential problems which result in less accurate coefficients, even in corrected Legendre polynomial coefficients in a high resolution spectral transform, if we prepare the coefficients with different iteration methods. And there are several different iteration methods as shown in Swarztrauber (1993), however, we selected one with an easy implementation for parallel computing but which requires resolving a machine underflow problem.

2. Accurate associated Legendre polynomial with very high resolution

There is an underflow problem in computing the Legendre base function for transform in the traditional three-item iteration method, which results in an error transformation for wave numbers larger than 1900 with double precision, and wave numbers larger than 900 with single precision. A method we call x-number has been implemented into the NCEP GFS to avoid this underflow problem, thus the transformation can be applied to any given high resolution up to several thousand wave numbers. The method is described in detail in Fukushima (2011), which we can paraphrase briefly here as follows:

Any real number, f, can be represented by one real number and one integer number with a big base as

$$f = xB^i$$

where x is the real number and i is an integer, and B is the base number. For single precision, B can be 2 to the power of 360, and for double precision, B can be 2 to the power of 960, which is big enough to take care of over- and underflow from machine limitations.

The main concept is that each real number has to be normalized its x values, so that any two x's numbers time together; the results cannot have over- or underflow. For double precision, B=2^960, as the normalized range for any real number is between 2^(-480) to 2^480. To prepare associated Legendre polynomial coefficients using the traditional iteration method in x-number, we first put the associated Legendre polynomial coefficient in x-number, as f=x and i=0, then normalize it before any multiplication, and then normalize it again before any iteration with multiplication, etcetera, during entire iteration procedure. After all the associated Legendre polynomial coefficients in x-number are obtained, they are represented back to a real number f by the formula with x, B, and i. If an underflow results, then we put in zero. There is no overflow in our associated Legendre polynomial preparation.

To test how x-number helps, we use the spectral transform utility in the NCEP library. First, we decide on a resolution and give it to all real parts of the spectral coefficients, then transform from the spectral coefficient to a physical grid, then transform from the physical grid values back to spectral coefficients. If the associated Legendre coefficients are correct, the resulting spectral coefficients after one complete spectral transform (from spectral to grid then grid to spectral), should be very close to the original value. Fig. 1 shows the absolute difference between the original value and value after one complete spectral transform without using x-number in base 10 logarithmic. There are incorrect transformed values above n>1500, the reason being the preparation of middle m values have underflow, which machine cannot present precisely, and the error accumulates through iterations from low value n to higher value n.

Fig. 2 is the same procedure as for Fig. 1 except it uses x-number to prepare associated Legendre polynomial coefficients for spectral transform. There is no error and accuracy is up to 6 or 7 digits. We can have even more accuracy by improving Gaussian weighting factor, not shown here.

3. Discussion

The Fukushima x-number method has been implemented into NCEP GFS for testing with resolutions of T574, T1534 and T3000 with success. The method concerns underflow on the multiplications only; there is no concern about overflow in associated Legendre polynomial coefficient preparation. And the x-number is used only for preparation; there is no change for spectral transform while using the Legendre polynomial coefficient, which is used as the real number, and not x-number in the model integration. The x-number was implemented into NCEP GFS for operational use in 2015.

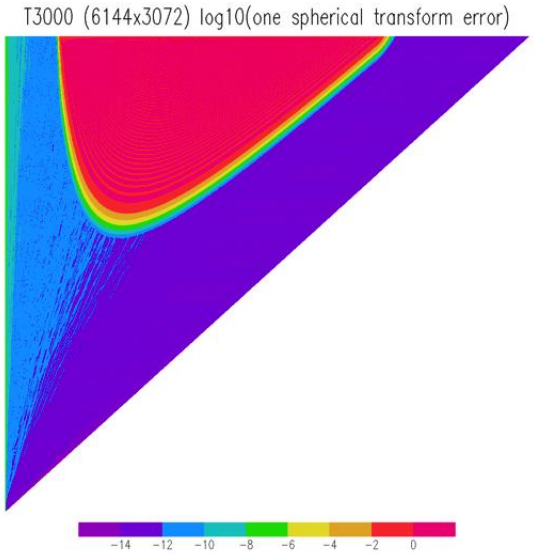

Fig. 1 log10 of the absolute difference between original spectral coefficient and after one complete spectral transform with traditional three-term iteration in T3000, which has linear Gaussian grids of 6144x3072.

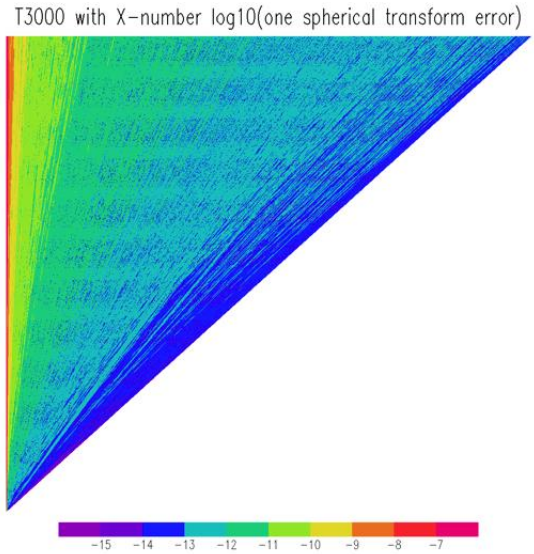

Fig. 2 The same as Fig. 1 except using x-number during iteration to prepare associated Legendre polynomial coefficients.

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