A finite element scheme for the vertical discretization in the semi-Lagrangian version of the ECMWF forecast model

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1 Introduction

The treatment of the vertical part of the gravity wave terms in the forecast equations by means of finite differences in the Lorenz arrangement of variables (hereafter FDL) was the least accurate part of the dynamics of the ECMWF model, being only first order in contrast to the higher order achieved by the spectral discretization in the horizontal and the 3-dimensional semi-Lagrangian advection. To improve its accuracy, a cubic finite element scheme (hereafter FE) for the vertical discretization has been developed and implemented recently in the operational version of the ECMWF analysis/forecast model. The scheme uses cubic B-splines as basis functions. No staggering of variables is required, making it ideally suited for use in conjunction with the semi-Lagrangian advection.

2 Vertical integral operator in finite element representation

In the semi-Lagrangian version of the ECMWF forecast model (see Ritchie et al. 1995) the only vertical operator used is the vertical integral operator.
The vertical integrals involved in the solution of the equations are: 1.) The vertical integral of the continuity equation from the top of the atmosphere to each of the model levels, which gives the vertical velocity needed by the semi-Lagrangian advection and the pressure vertical velocity needed in the thermodynamic equation. 2.) The integral of the continuity equation from the top of the atmosphere to the surface, which gives the tendency of the surface pressure. 3.) The integral of the hydrostatic equation from the surface to each of the model levels.
Integrals starting at the surface can be computed from the corresponding integrals starting at the top of the atmosphere by subtracting the integral from the top of the atmosphere to the surface. Consequently, the only vertical operator needed is the operator "integral from the top of the atmosphere to each of the model levels and to the surface".
This vertical integral operator has been evaluated in finite element representation by using cubic B-splines as basis functions with compact support (Prenter, 1975, and references therein) and by applying the Galerkin method. We incorporated the transformations to spline space and back to physical space into the operator to obtain a matrix which operates in physical space, i.e. when applied to a function given on model levels (plus appropriate boundary conditions), the operator yields the values of the integral of this function on model levels.

3 Results

The new FE scheme has been tested extensively in runs with the dynamical core version of the model as well as with the full model in data assimilation and forecast mode. The main benefits we have found are:
1.) The vertical linear gravity eigenmodes computed with the FE scheme are more similar than the FDL ones to the eigenmodes obtained with finite differences in Charney-Phillips arrangement. The latter arrangement of variables is ideally suited to the treatment of the gravity wave terms in the forecast equations and is free of the spurious computational mode in the vertical present in the

FDL, but it is more difficult to use in conjunction with semi-Lagrangian advection due to the staggering of temperature and winds. The FE scheme improves on the treatment of the gravity wave terms while retaining the advantage for the semi-Lagrangian scheme of not having staggered variables.

2.) The FE scheme reduces substantially the amplitude of the small scale vertical structures produced in divergence (mainly in the tropical stratosphere) with the FDL scheme.

3.) This noise reduction in divergence, together with the improved vertical integration of the continuity equation with the FE integral operator, leads to a more accurate computation of the vertical velocity which in turn improves the semi-Lagrangian advection in the vertical. As a consequence, tracer conservation in the semi-Lagrangian model is improved with the FE scheme. Fig. 1 shows the percent change in global ozone mass in two ensembles of 60-day forecasts (with the ozone chemistry switched off) run with the FE scheme (dashed line) and with the FDL scheme (control runs). With the FE scheme the lack of conservation is reduced by about 50%.

4.) In data assimilation, a better fit of the first guess to radiosonde data in the lower stratosphere is found with the FE scheme, resulting in the use of more observations in the analysis.

5.) The FE scheme also gives a substantial improvement in stratospheric skill scores, notably in RMSE of temperature in the tropics.

4 References

Prenter, P. M., 1975: Splines and variational methods, *John Wiley& Sons.*

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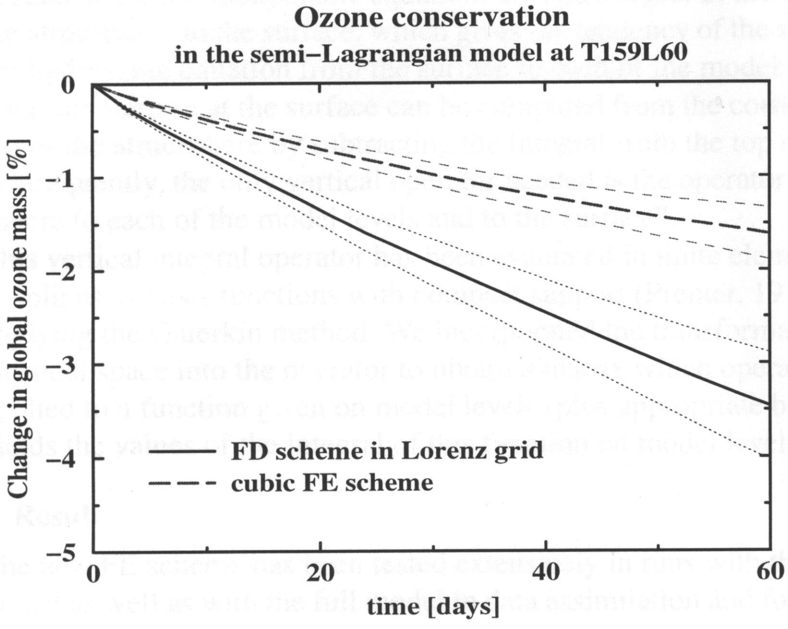

Fig. 1 Percent change of global ozone in two ensembles of 60-day forecasts using the cubic finite element scheme (dashed line) and the finite difference scheme in Lorenz arrangement (control) (full line). The thick lines are the mean and the thin lines give an indication of the spread (mean± stdev).