Use of potential vorticity as a control variable in a 4DVAR assimilation system.

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This note describes experiments to use control variables based on potential vorticity and imbalance to replace vorticity and unbalanced height in the ECMWF data assimilation system. Standard geostrophic adjustment theory shows that the balanced information is carried by the vorticity when the aspect ratio is greater than *f/N*, and by the geopotential otherwise. The present formulation is based on Parrish and Derber (1992) and uses vorticity as the balanced variable, with a linearly balanced height calculated from it and associated temperature and surface pressure deduced statistically. The remaining temperature and surface pressure form the unbalanced variable.

The new formulation is illustrated as follows. Consider the linearised spectral equations for a single zonal wavenumber:

$$\begin{aligned}
\frac{\partial\zeta}{\partial t} + \mathbf{F}D &= \mathcal{Z} \\
\frac{\partial T}{\partial t} + \tau_r D &= \mathcal{T} \\
\frac{\partial}{\partial t}\ln p_{surf} + \nu D &= \mathcal{P} \\
\frac{\partial D}{\partial t} + \nabla^2(\gamma_r T + \sigma_r \ln p_{surf}) - \mathbf{F}\zeta &= \mathcal{D}
\end{aligned} \tag{1}$$

Here, ζ, D, T, p_{surf} represent the vertical component of vorticity, the horizontal divergence, and the temperature and surface pressure. $\mathbf{F}$ is a matrix relating different meridional wavenumbers (the Coriolis parameter), Greek letters represent matrices connecting model levels, and calligraphic letters represent nonlinear terms. The new balance operator can be written as follows for a single vertical mode with gravity wave speed c_n^2:

$$\begin{aligned}
(c_n^2 - F\nabla^{-2}F)\zeta_{bn} &= Q_n = c_n^2\zeta_n - F\phi_n \\
(F^2 - c_n^2\nabla^2)\phi_{un} &= -c_n^2 R_n = -c_n^2(\nabla^2(\gamma_r T + \sigma_r \ln p_{surf}) - F\zeta) \\
\nabla^2\phi_{bn} &= F\zeta_{bn} \\
T_n &= \tau_r c_n^{-2}\phi_{bn} + T_{un} \\
\ln p_{surfn} &= \nu c_n^{-2}\phi_{bn} + \ln p_{surfun} \\
\phi_{un} &= \tau_r T_{un} + \nu \ln p_{surfun} \\
\zeta_n &= c_n^{-2}\phi_{un} + \zeta_{bn}
\end{aligned} \tag{2}$$

Here the suffices *b* and *u* refer to balanced and unbalanced variables. Q is the potential vorticity appropriate for the linearised equations (1). R is the imbalance. The difficulty with this formulation occurs for modes when $\mathbf{F}$ has a small eigenvalue and c_n^2 is small. This prevents the use of Q_n, R_n directly as control variables. It is found best to use ζ_b, T_u, $\ln p_{surfu}$ as the control variables. Even then, it is necessary to regularise the change of variable either by adding a small term to the left hand side of the first two equations in (2) or by excluding certain modes from the change of variable altogether.

The scheme was still not successful when used with the currently operational form of the ECMWF model on the Lorenz vertical grid. This is because there are many small values of c_n^2 . Recently a new vertical formulation has been developed using finite elements, Untch and Hortal (2002), which should become operational early in 2002. This has larger values of c_n^2 for small vertical scales. Results are shown for a 35 day parallel run using the finite-element version of the model and the new balance operator. Each set of forecasts was verified against their own analyses.

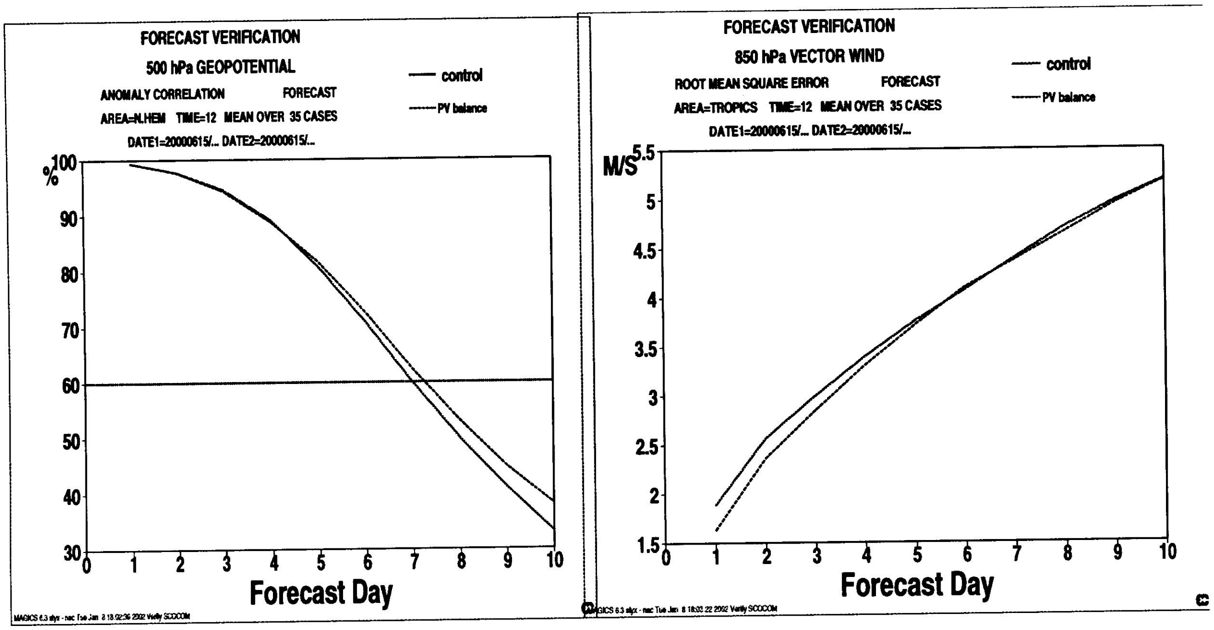

Figure 1 . Left panel: Anomaly correlations for 500hpa forecasts, Northern hemisphere. Right panel: r.m.s. errors of 850hpa wind forecasts for the tropics.

The results show a useful positive impact in the Northern hemisphere. There is also a large positive impact on the tropical wind scores. The latter is because the analyses are significantly smoother, though they verify equally well against observations.

Parrish,D.F. and Derber,J.C. (1992) The National Meteorological Center's spectral statistical interpolation analysis system. *Mon. Weather Rev.*, **120**, 1747--1763

Untch,A. and Hortal,M. (2002) A finite element scheme for the vertical discretisation in the semi-Lagrangian version of the ECMWF forecast model. In preparation.