Theoretical examination of semi-implicit time-scheme for nonhydrostatic NWP

Pierre Bénard*, Jozef Vivoda+, Petra Smolikova†, Jan Masek+

* *Centre National de Recherches Météorologiques, Météo-France, Toulouse, France*

+ *Slovak Hydro-Meteorological Institute, Bratislava, Slovakia*

† *Czech Hydro-Meteorological Institute, Prague, Czech Republic*

Severe instabilities were experienced in Aladin-NH (the NH research version of the cooperative NWP LAM model Aladin) when pushing the time-step to values compatible with the implementation of a semi-implicit semi-Lagrangian (SISL) transport scheme in NWP mode. This instabilities were found to be linked to the SI scheme, and to have some similarity with the non-linear instabilities analyzed by Simmons et al., 1978 (SHB78) in the Hydrostatic Primitive Equation (HPE) system. Hence a theoretical study to investigate (and possibly remedy to) this behaviour in the Euler Equations (EE) system was undertaken. The instabilities were successfully reproduced in a framework compatible with theoretical analysis, i.e. linear, resting and isothermal atmosphere. All the analyses are NOT space-discretized, hence the instabilities cannot be suspected to come from some deficient particular choices in the space discretization of the model. Conversely, the framework allows to state on the relevance of other stategic choices: set of prognostic variables, vertical coordinate, time-discretization.

The first simple framework where this instability could be reproduced theoretically was the SHB78 one: when the actual temperature $\overline{T}$ deviates from the SI reference one T^*, for a flat-terrain domain. In this framework, it was proved that the problem is intrinsically due to the original choice for the set of the NH prognostic variables in Aladin-NH, described in Bubnova et al., 1995 (BHBG95). A new set of NH prognostic variables which removed this problem was proposed:

$$\mathcal{P} = \frac{p-\pi}{\pi}$$
$$\mathsf{d} = -g\frac{p}{mRT}\frac{\partial w}{\partial \eta}$$

where p is the pressure, π is the hydrostatic pressure, η is the hybrid hydrostatic-pressure based vertical coordinate (cf: Laprise, 1992), $m = (\partial\pi/\partial\eta)$, and other notations are standard.
With this solution, the stability was proved to be very similar to the one obtained with height-based hybrid vertical coordinates, hence the hydrostatic-pressure-based coordinate has no structural weakness with this respect, compared to height-based coordinates.
It is worth to note that such severe stability problems linked to the choice of the set of prognostic variables were not present in the HPE system, for which the stability of the SI scheme is found to be not very sensitive to the choice of prognostic variables. Another salient result is that even when an optimal choice of prognostic variable is done, the width of the stability domain in the examined context is significantly reduced from HPE system to EE system. The stability domain is $0 < \overline{T} < 2T^*$ for the HPE system and $T^*/2 < \overline{T} < 2T^*$ for the EE system. EE system with SI time-discretization is thus intrinsically less robust than HPE system.

However, in presence of orography, the above proposal was still experienced to induce instabilities, and a framework to better understand the effects of orography hence needed to be examined.

The simple framework where this new instability could be reproduced theoretically consisted in the same as above except that a uniform slope was introduced. This is physically consistent because of the isothermal character of the considered atmosphere, and results in a uniformly slanted coordinate system, which in turn induces additional explicitely-treated terms proportional to the slope in this linear context. In this framework, it was proved that with the above-mentionned proposal ($\mathcal{P}$, d) for the set of NH prognostic variables, the SI scheme turned unstable again. Finally, a new set of NH prognostic variables which alleviated this problem was proposed:

$$\mathcal{P} = \frac{p - \pi}{\pi}$$
$$\mathrm{d}' = D_3 - \nabla V$$

where D_3 is the 3-dimensional divergence, V is the horizontal wind, and ∇ is the derivative operator along constant η-surfaces. The effect of this new variable is to allow a complete semi-implicit treatment for the acoustic term D_3 which arises due to the compressibility in the pressure and temperature equations.
The analyses showed that the new d$'$ variable was very beneficial for the stability. However, in contrast with the flat-terrain case discussed above, a residual instability remains even with the new d$'$ variable. This residual instability of course increases with the slope and the spatial resolution.
This second problem has its exact counterpart in height-based systems, and a similar alleviating solution is traditionnally adopted in models using this vertical coordinate: the vertical velocity w is replaced as a prognostic variable, by a new variable W which also includes the cross term induced in the 3-dimensional divergence by the slanted metrics.

The conclusions that we draw from these analyses can then be formulated as follow: in presence of steep orography and high resolutions, even with a relevant choice of prognostic variables, the SI system as traditionally formulated could be not robust enough to support fully compressible systems. Hence more robust schemes must be examined. The theoretical framework presented here allows to examine the behaviour of such alternative temporal-schemes.

References

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