Spectrum of Linearized Operator of Atmospheric Circulation Hydrodynamic Model: Method of Evaluation and Applications

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Eigen vectors of a linearized operator of an atmospheric circulation hydrodynamic model are well known in meteorology as singular vectors (SV's). A method of evaluation of these vectors is known as a breeding-method. The concern to SV's has arisen in connection with usage of an ensemble of hydrodynamic forecasts (Tracton and Kalnay, 1993).

Consider a method of SV's evaluation. Let Δt be a short time interval, $\boldsymbol{X}_0$ be an initial state (initial data) in an instant t_0, $\boldsymbol{Y}_0$ be a result of integration of a model on a time interval $[t_0,t_0+\Delta t]$. Let $\Delta\boldsymbol{X}_0$ be an initial perturbation and let the norm ($|\Delta\boldsymbol{X}_0|=<\Delta\boldsymbol{X}_0,\Delta\boldsymbol{X}_0>^{0.5}$, $<*,*>$ is a scalar product) of this perturbation be a peer to the standard of errors of measuring and analysis. By $\boldsymbol{Y}_1$ denotes the result of integration of a model on the time interval $[t_0,t_0+\Delta t]$ from an initial state $\boldsymbol{X}_0+\Delta\boldsymbol{X}_0$, we have $\Delta\boldsymbol{X}_1=\Delta\boldsymbol{Y}_0|\Delta\boldsymbol{X}_0||\Delta\boldsymbol{Y}_0|^{-1}$, where $\Delta\boldsymbol{Y}_0=\boldsymbol{Y}_1-\boldsymbol{Y}_0$. Obviously $\Delta\boldsymbol{Y}_0\approx\mathbf{L}\Delta\boldsymbol{X}_0$, where $\mathbf{L}$ is an operator of a hydrodynamic model linearized on the time interval $[t_0,t_0+\Delta t]$.

Using this iterative method $\Delta\boldsymbol{X}_{k+1}=\Delta\boldsymbol{Y}_k|\Delta\boldsymbol{X}_0||\Delta\boldsymbol{Y}_k|^{-1}$, where $\Delta\boldsymbol{Y}_k=\boldsymbol{Y}_{k+1}-\boldsymbol{Y}_0$, $\boldsymbol{Y}_{k+1}$ is a result of integration of a model on the time interval $[t_0,t_0+\Delta t]$ from an initial state $\boldsymbol{X}_0+\Delta\boldsymbol{X}_k$ ($\Delta\boldsymbol{Y}_k\approx\mathbf{L}\Delta\boldsymbol{X}_k$), we shall receive the first SV. The ratio $<\Delta\boldsymbol{Y}_k,\Delta\boldsymbol{X}_k>/<\Delta\boldsymbol{X}_k,\Delta\boldsymbol{X}_k>$ tends to the first (maximum) eigen value λ_1.

For deriving remaining SV's it is necessary to use orthogonalization. For introduction of metric relations the energy scalar product will be used. Scaleted SV's with $\lambda_i>1$ will be used as the perturbation of a hydrodynamic model in the ensemble of forecasts and ensure a maximum ensemble scatter (or variance) (Pichugin et. al., 1998).

The surveyed method is identical to a well-known method of linear algebra. It is R. von Mises method of iterative evaluation of eigen vectors and eigen values of a matrix.

Consider two SV-spectrum applications.

1) All SV's are populations of fields of perturbations. We can consider the perturbation of one of the field, for example H_{500}, we shall denote it as $\boldsymbol{F}(\varphi,\psi)$, where φ and ψ are also geographical coordinates. The spatial distribution of $\boldsymbol{F}$ demonstrates where the most sensitive to errors of measuring and analysis and the most related to dynamic instability geographical bands are posed. Where the absolute values of $\boldsymbol{F}$ are higher there measuring errors influence most hardly the result of integration of a model.

If the major sampling SV's ($\boldsymbol{F}$) is accumulated, then using the sampling principal components (of $\boldsymbol{F}$) regression and regression experiments design methods we can realize selection of the most informative geographical points related to the dynamic instability (Pichugin and Pokrovsky, 1992).

2) SV-spectrum $\{\lambda_i\}$ is a performance of instability of an initial state. For example a spectral radius (λ_1) is a good statistical predictor (regressor) for dynamic forecast skill (of error of the forecast). On experimental data, λ_1 correlates with a mean square error of a hydrodynamic monthly forecast of the field H_{500} (r=0,4 with a significance level α=0.001, after elimination of seasonal effects r=0,24 with a significance level α=0.05). For statistical prediction of an error of hydrodynamic forecast in the regression equation to this predictor it is possible to add the other

predictors describing instability of hydrodynamic forecast (or an initial state), for example estimations of variances of the field H_{500} in different geographical points obtained by hydrodynamic forecast ensemble (and see 1-st application).

References

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